GRAVITATION Volume 3 Issue 1 1997
Journal on gravitational physics issues page 71—77

PROBLEM QUESTIONS OF THE SPECIAL RELATIVITY

SUPERLUMINAL VELOCITIES, IMPROPER ROTATIONS AND THE CHARGE SYMMETRY

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In this paper we consider the possibility of direct observation of motions with superluminal velocities. We show that the invariance of the interval in the special relativity theory makes observations of superluminal motions impossible. However, the symmetry of speeds in the classical Lorentz transformations with reference to that of light is a physical reality and manifests itself as the charge symmetry (reverse of time) and/or the symmetry of reflection. CPT-invariance is a natural sequence of this symmetry, which corresponds to improper 4-dimensional rotations in geometry.

Existence of tachyons, hypothetical particles which travel in vacuum at superluminal velocities, i.e. faster than light, is assumed to be in no contradiction with the special relativity theory (SRT) that bans only transitions across the light barrier. Lack of evidence of their existence has not yet caused any difficulty in theory, nevertheless hundreds of papers deal with the velocity symmetry problem (a detailed list of references is presented in Ref. 1). Tachyons are generally believed to have an imaginary mass at rest (Ref. 2) so that their relativistic mass at $|v| > c$ be a real value tending to be zero at $|v| \to \infty$, the momentum tending to become a constant value (Ref. 3). Under certain conditions superluminal motions would cause reversals of causality in time, which fact makes those who argue for the tachyon hypothesis either postulate the relativity of division of correlated phenomena into causes and effects (Ref. 4), or discard SRT in the $|v| > c$ region (Ref. 5), or ban detectability of tachyons in macroscopic scale phenomena and restrict it by supermicroscopic space and time scales, where the reversal of causalities in time would not break the second law of thermodynamics, probably (Ref. 6).

The extrapolation of SRT to the $|v| > c$ region by adding the reinterpretation postulate or the concept of "retarded causality" (Ref. 3, 7) to SRT, the square of interval being changed by its sign with the formal transition of the light barrier, means acception of three time-like dimensions and one space-like dimension of the tachyon world, unless both sub- and superluminal worlds are $(3 + 3)$-dimensional (Ref. 3). The author of Ref. 8 tried to eliminate imaginary values resulting from superluminal transformations by introducing of a subluminal reference frame, where a space axis coincided with the world line of a tachyon ("stop-time" of the tachyon).

All these attempts not only legalize the inversion of causalities under the known conditions, but also mean acceptance of umbiguity of the "tachyon — antitachyon" and "particle — antiparticle" relations, both dependent on the choice of an inertial reference frame. The author of Ref. 9 had suggested an interesting argumentation for the possibility of observation of superluminal motions never breaking the causality principle, which, nevertheless, required introduction of a hypothetical medium (hyperfluid) for propagation of superluminal signals.

The present paper is aimed to clarify the conclusions presented in the previous paper (Ref. 10) revealing the author's viewpoint on consistent application of SRT to the superluminal region and which permit, to his mind, consider objects' properties in the $|v| > c$ region without any complementation to classical SRT. The basic statements of Ref. 10 are as follows.

The consequent application of SRT will be defined as extrapolation to the superluminal region of: a) the postulate of the equality of rights of all inertial frames of reference in terms of description of physical phenomena, in particular, in three dimensional space with one-dimensional time, squares of space-like intervals having opposite signs as compared to squares of time-like ones; we will view the latter as negative in every reference frame; b) applicability of Lorentz transformations to differential physical quantities.

Let the coordinates of two events in a certain inertial frame (further referred to as the stationary or fixed frame) be (icT, X) and $(icT + ic\,dT, \mathbf{X} + d\mathbf{X}$, i.e. square of the interval $i\,dS$ between the events in Minkovski space-time is given by the equality:

$$(i\,dS)^2 = -(c\,dT)^2 + |d\mathbf{X}|^2 = (ic\,dT)^2 + |d\mathbf{X}|^2. \tag{1}$$

The square of the same interval in a frame of reference that moves about the first frame with a constant velocity (further referred to as the moving frame) is expressed as follows:

$$(i\,dS)^2 = \left(ic\frac{dT - v\,dX_1/c^2}{(1-v^2/c^2)^{1/2}}\right)^2 + \left(\frac{dX - v\,dT_1}{(1-v^2/c^2)^{1/2}}\right)^2 + dX_2^2 + dX_3^2 \equiv$$
$$\equiv (ic\,dT')^2 + (dX_1')^2 + (dX_2')^2 + (dX_3')^2, \tag{2}$$

where dX_1 and dX_1' are the projections of the interval on the direction of motion of the moving frame in the stationary and moving frames respectively; $dX_2 = dX_2'$, $dX_3 = dX_3'$ are the projections of the interval on mutually orthogonal directions which are orthogonal to the direction of motion of the moving frame; $ic\,dT'$ is the projection of the interval on the time axis of the moving frame.

In both frames of reference the interval has one time-like projection expressed in Equations (2) and (3) by an imaginary number and three space-like projections expressed in these equations by real numbers. Two real-number projections which are perpendicular to the direction of the relative motion are not transformed, while the real-number projection parallel to the direction of the relative motion and the imaginary-number (time-like) projection are given Lorentz transformations at the transition from the stationary frame to the moving one.

Referring to the case of superluminal velocities we find that after the substitution of $|v| > c$ in the right-hand part of Eq. (3) three projections of the interval, viz. $ic\,dT'$, dX_2' and dX_3', remain real values, and there is still one imaginary projection dX_1'. Proceeding from the equality of inertial frames of reference we hold that in the superluminal frames as well the interval has three real-unit-measurable space-like projections, two of them, those perpendicular to the direction of motion $dX_2' = dX_2$ and $dX_3' = dX_3$, being untransformable, and one imaginary-unit-measurable time-like projection. The third space-like projection X_1'', parallel to the direction of the relative motion, is necessarily the only remaining real value $ic\,dT'' = (dX_1 - dTc^2/v)(1-c^2/v^2)^{1/2}$ while the only time-like projection $ic\,dT''$ is the only imaginary value $dX_1' = -ic(dT + dX_1/v)/(1-c^2/v^2)^{1/2}$

and transformations of the coordinates in the case under discussion prove to be identical to Lorentz transformations to within the algebraic sign of the time-like projection, with provision for substitution $v \to c^2/v$.

H. Poincare (Ref. 11) was the first to indicate identity of Lorentzian transformations (with an imaginary time coordinate) and the transformations of a rotation of the axes icT and X_1 in the plane by an imaginary angle φ' defined by the relationships

$$ic\,dT' = ic\,dT\cos\varphi' + dX_1\sin\varphi',$$
$$dX_1' = dX_1\cos\varphi' - ic\,dT\sin\varphi',$$
$$\varphi' = \frac{1}{i}\ln(\cos\varphi' + i\sin\varphi') = \frac{1}{i}\ln\left(\frac{1}{(1-v^2/c^2)^{1/2}} + i\frac{v/ic}{(1-v^2/c^2)^{1/2}}\right) = \tag{3}$$
$$= \frac{1}{2i}\ln\frac{1+v/c}{1-v/c}.$$

The new transformations of the coordinates obtained for the $|v| > c$ case are a particular case of coordinate transformations at a four-dimensional rotation by a complex angle φ' whose real part equals the odd number of $\pi/2$:

$$\varphi' = \frac{1}{2i}\ln\left(-\frac{1+c/v}{1-c/v}\right) = \frac{1}{2i}\left(\ln(-1) + \ln\left(\frac{1+c/v}{1-c/v}\right)\right) =$$
$$= \frac{1}{2i}\left((2n-1)\pi i + \ln\left(\frac{1+c/v}{1-c/v}\right)\right) = (2n-1)\frac{\pi}{2} + \frac{1}{2i}\ln\frac{1+c/v}{1-c/v} \tag{4}$$

with an inversion of either time (at odd values of n) or the coordinate in the direction of the motion (at even values of n).

The observed velocity of motion of the moving frame of reference is determined in all the cases by the following procedure:

a) an elementary interval is considered, which has only one non-zero projection, specifically a time-like projection (intrinsic time interval), in the moving frame of reference;
b) transformations of the rotation by the angle φ' (Eq. 3) are used to determine the time-like projection $ic\,dT$ and the projection dX_1 on the direction of the relative motion in the fixed frame of reference;
c) the observed velocity w is found as the ratio of the space-like projection dX_1 to the elementary time interval dT.

In the $|v| > c$ case we have:

$$w = \left.\frac{dX_1}{dT}\right|_{dX_k''=0} = \left.\frac{dX_1}{dT}\right|_{ic\,dT'=dX_2=dX_3=0} = \left.\frac{dX_1}{dT}\right|_{dX_1 - dTc^2/v=0} = \frac{c^2}{v}. \tag{5}$$

At $|v| > c$ the 3-vector v loses the sense of the observed relative velocity while maintaining its role of one of the parameters which determine the mutual orientation of the coordinate axes of the two inertial frames of reference that have imaginary time axes. As for the observed relative velocity w, it is found to be subluminal both at $|v| < c$ and at $|v| > c$. The analogous value in Ref. 8 is not the observed velocity of a tachyon, but the observed velocity of a clock of a subluminal frame of reference for which the tachyon's world line serves as one of the space axes. We expect that development of the suggested approach would show that the change of v into the phase velocity of a particle moving with w (i.e. that at $|v| > c$ the group and phase velocities change roles) is not incidental.

The angles (4) do not exhaust the set of possible planar rotations of icT and X. Proceeding from the notions of equality of inertial frames of reference, in particular equality of the frames rotated by the angle (4) about the fixed one and those with $\mathrm{Re}\,\varphi' = 2\pi n$ ("hypophotic" ones), we must insist on existence of inertial frames of reference rotated by the angle (4) respective to the "hyperphotic" systems; the angle of rotation of these "new" frames respective to the fixed one will be, taking into account the substitutions of c/v for w_0/c and w/c in Eq. 4,

$$\varphi' = \left((2n-1)\frac{\pi}{2} + \frac{1}{2i}\ln\frac{1+w_0/c}{1-w_0/c}\right) + \left((2m-1)\frac{\pi}{2} + \frac{1}{2i}\ln\frac{1+w/c}{1-w/c}\right) =$$
$$= (n+m-1)\pi + \frac{1}{2i}\ln\frac{1+(1/c)(w_0+w)/(1+w_0w/c^2)}{1-(1/c)(w_0+w)/(1+w_0w/c^2)} = k\pi + \frac{1}{2i}\ln\frac{1+w_s/c}{1-w_s/c}, \quad (6)$$

where w_0 is the observed velocity of the "hyperphotic" frame and w is the velocity of the "new" frame observed from the "hyperphotic" one.

The observed velocity $w_s = (w_0+w)/(1+w_0w/c^2)$ is proved to satisfy the classical rule of composition of velocities, and since the absolute values of the velocities w_0 and w do not exceed the velocity of light, the observed velocities of the frames of reference which are rotated by the angle (6) about the fixed frame do not exceed the velocity of light. Thus the objects formally corresponding, from the classical SRT viewpoint, to tachyons are always observed as moving with subluminal velocities. The question put in Ref. 12: "Are tachyons faster-than-light particles?" — meets a definitely negative answer here. At even values of k the coordinate transformations coincide with Lorentz transformations, and at odd values they differ by the signs of the time-like and space-like projections of the interval on the direction of the relative motion (because of the inversion of $\cos\varphi'$ and $\sin\varphi'$ with φ' changed to the odd number of π).

The existence of four different forms of transformation, viz.

$$\begin{aligned} dT^{(l)} &= (-1)^{[l/2]}\frac{dT-(w/c^2)dX_1}{(1-w^2/c^2)^{1/2}}, \\ dX_1^{(l)} &= (-1)^{[(l-1)/2]}\frac{dX_1 - w\,dT}{(1-w^2/c^2)^{1/2}}, \\ dX_2^{(l)} &= dX_2, \\ dX_3^{(l)} &= dX_3, \end{aligned} \quad (7)$$

(where $l = 1,2,3,4$, and [ ] denotes integer part), that correlate the coordinates in frames of reference moving with the same observed velocity w and the coordinates in a fixed frame enables us to say that there are four different possible states of a frame of reference and consequently of a material body, which are:

— the state with $l = 1$ that corresponds to familiar Lorentz transformations with the "+" signs in the right-hand parts of Eq. 7;

— the state with $l = 2$ which is characterised by inversion of time with respect to Lorentz transformations;

— the state with $l = 3$ which is characterised by inversion of time and by a mirror-reflected spatial structure (transformations with the "−" signs in the right-hand parts of Eq. 7);

— the state with $l = 4$ which is characterised by a mirror-reflected structure with respect to the state with $l = 1$.

Within each state the body can have different values of the observed relative velocity w and different integer orientations in the three-dimensional space.

The analysis made with respect to transformations of projections of the invariant interval can be similarly done for tensors of differential quantities of any dimensionality.

Let us note that in Minkowski's time-space with an imaginary time axis the differences between transformations of covariant and countervariant tensors exist no longer; introduce the notation $icT = X_0$, and the corresponding tensor components we will mark with the zero subscript; Lorentz-transformed components will be primed. Thus the component of any tensor of an arbitrary order n has the following forms in the states $l = 1,2,3,4$:

$$T^{(l)}_{\alpha_1,\ldots,\alpha_n}(w) = (-1)^{k_l}T'_{\alpha_1,\ldots,\alpha_n}, \quad (8)$$

where $k_1 = 0$; k_2 is the number of zero subscripts of the tensor; k_3 is the total number of zero and unity subscripts of the tensor; k_4 is the number of unity subscripts of the tensor. Assuming that values of the density type are transformed in accordance with SRT and that the above analysis for coordinates is applicable to them, we shall obtain the following for the observed mass m, electric charge q and angular momentum J in the X_2, X_3 plane perpendicular to the observed velocity:

a) the mass (the scalar component of the 4-momentum):

$$m = \iiint\limits_{X_1X_2X_3} T_{00}\,dX_1dX_2dX_3 =$$

$$= \begin{cases} \displaystyle\iiint\limits_{X_1^{(l)}X_2^{(l)}X_3^{(l)}} \frac{T^{(l)}_{00}}{1-w^2/c^2}[dX_1^{(l)}(1-w^2/c^2)^{1/2}]dX_2^{(l)}dX_3^{(l)} = \\ \displaystyle= \frac{m_{\text{int.}}}{(1-w^2/c^2)^{1/2}}, \quad l = 1,2, \\ \displaystyle\iiint\limits_{X_1^{(l)}X_2^{(l)}X_3^{(l)}} \frac{T^{(l)}_{00}}{1-w^2/c^2}[-dX_1^{(l)}(1-w^2/c^2)^{1/2}]dX_2^{(l)}dX_3^{(l)} = \\ \displaystyle= -\frac{m_{\text{int.}}}{(1-w^2/c^2)^{1/2}}, \quad l = 3,4, \end{cases} \quad (9)$$

where T_{00} is the component of the tensor of the energy-momentum density that does not change the sign in different states because of the even number of zero subscripts; $m_{\text{int.}}$ is the intrinsic mass of the body:

$$m_{\text{int.}} = \iiint\limits_{X_1^{(l)}X_2^{(l)}X_3^{(l)}} T^{(l)}_{00}\,dX_1^{(l)}dX_2^{(l)}dX_3^{(l)};$$

b) the electric charge:

$$icq = \iiint\limits_{X_1X_2X_3} j_0\,dX_1dX_2dX_3 =$$

$$= \begin{cases} \displaystyle\int\!\!\int\!\!\int\limits_{X_1^{(l)}X_2^{(l)}X_3^{(l)}} \frac{\pm j_0^{(l)}}{(1-w^2/c^2)^{1/2}} \left[\pm dX_1^{(l)} \left(1-\frac{w^2}{c^2}\right)^{1/2}\right] dX_2^{(l)} dX_3^{(l)} = \\ = icq_{\text{int.}}, \quad l=1,3 \\ \displaystyle\int\!\!\int\!\!\int\limits_{X_1^{(l)}X_2^{(l)}X_3^{(l)}} \frac{\mp j_0^{(l)}}{(1-w^2/c^2)^{1/2}} \left[\pm dX_1^{(l)} \left(1-\frac{w^2}{c^2}\right)^{1/2}\right] dX_2^{(l)} dX_3^{(l)} = \\ = -icq_{\text{int.}}, \quad l=2,4, \end{cases} \tag{10}$$

where j_0 is the time-like component of the 4-density of the current, changing the sign simultaneously with $X_0 = icT$; $q_{\text{int.}}$ is the intrinsic electric charge:

$$icq_{\text{int.}} = \int\!\!\int\!\!\int\limits_{X_1^{(l)}X_2^{(l)}X_3^{(l)}} j_0\, dX_1 dX_2 dX_3.$$

c) the angular momentum:

$$J_{23} = \int\!\!\int\!\!\int\limits_{X_1 X_2 X_3} (X_2 T_{30} - X_3 T_{20})\, dX_2 dX_2 dX_3 =$$

$$= \int\!\!\int\!\!\int\limits_{X_1^{(l)}X_2^{(l)}X_3^{(l)}} \frac{\pm(X_2^{(l)}T_{30}^{(l)} - X_3^{(l)}T_{20}^{(l)})}{(1-w^2/c^2)^{1/2}} \left[\pm dX_1^{(l)} \left(1-\frac{w^2}{c^2}\right)^{1/2}\right] dX_2^{(l)} dX_3^{(l)} =$$
$$= J_{23\text{int.}}, \quad l=1,3,$$

$$J_{23} = \int\!\!\int\!\!\int\limits_{X_1^{(l)}X_2^{(l)}X_3^{(l)}} \frac{\mp(X_2^{(l)}T_{30}^{(l)} - X_3^{(l)}T_{20}^{(l)})}{(1-w^2/c^2)^{1/2}} \left[\pm dX_1^{(l)} \left(1-\frac{w^2}{c^2}\right)^{1/2}\right] dX_2^{(l)} dX_3^{(l)} = \tag{11}$$
$$= -J_{23\text{int.}}, \quad l=2,4,$$

where X_2, X_3 are the coordinates in the frame of reference of the mass centre; T_{20}, T_{30}, the components of the tensor of the energy-momentum density that change the sign simultaneously with X_0 because of the odd number of zero subscripts; $J_{23\text{int.}}$, the intrinsic angular momentum in the frame of reference of the mass centre:

$$J_{23\text{int.}} = \int\!\!\int\!\!\int\limits_{X_1^{(l)}X_2^{(l)}X_3^{(l)}} (X_2^{(l)}T_{30}^{(l)} - X_3^{(l)}T_{20}^{(l)})\, dX_2^{(l)} dX_3^{(l)}.$$

It follows from Eq. 10, 11, 12 that a particle in the states 2 and 3 has the opposite-sign charge-to-mass ratio as compared to the states 1 and 4, being therefore observed either as a particle (e.g. in the states 1 and 4 or as an anti-particle (in the states 2 and 3, respectively), while in different states with identical signs of the q/m ratio it is observed with opposite-sign angular momenta.

The case of an arbitrary position of Poincare's plane of a four-dimensional rotation of axes and the case of composition of such rotations in different planes that comprise an imaginary time axis can be reduced to one of the above-considered transformations with $l = 1,2,3,4$, depending on the residue of the number of real angles $\pi/2$ to the

modulus 4 in the total rotation angle, with the help of purely spatial rotations which do not affect the substance of the matter.

Thus the invariance of the interval under transformations of the coordinates and the equality of inertial frames of reference in the pseudo-Euclidean space-time permit the following conclusions:

a) it is impossible to observe hyperphotic velocity motions;

b) the symmetry of the parameter v about the "light barrier" allowed by SRT is nothing more than the symmetry of reflection, i.e. the symmetry of inversion of time and/or mirror reflection of the spatial axis parallel to the "fictitious" velocity vector **v**; in other words, this symmetry complements Lorentz group with improper rotations, or rotations with reflection;

c) a physical manifestation of this symmetry is the symmetry of particles and anti-particles, i.e. the charge symmetry;

d) the CPT-invariance follows from the conclusions b) and c);

e) with the formal transition of the light barrier the group velocity of a particle becomes the phase velocity of its "mirrored" state, i.e. the observed group velocity of particles corresponding to "superluminal" states is always less than that of light.

Naturally, the conclusions of this paper are valid with the validity of its initial assumptions only any of which may be discarded once (if an actual tachyon is discovered all of them would be discarded), therefore the author does not mean to finally close the search for actually superluminal motions.

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